Adaptive Sampling for Transient Signal Detection in the Presence of Missing Samples

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Abstract—The problem of interest is the detection of transient signals in additive white Gaussian noise (AWGN) in the presence of missing signal observations (samples). Specifically, a fusion center aims at detecting the presence of transient signals by collecting measurements from individual sensors through erasure channels. Under the assumption that the fusion center can control the sampling procedure through a feedback channel, a strategy is proposed to adapt the sampling rate in response to sample missing with the goal of achieving accurate and timely decisions with the minimum communication cost measured by sampling rate. The proposed strategy is flexible in that it can be configured to suit different performance requirements. Compared with fixed-rate sampling, the proposed strategy achieves better tradeoff between Quality of Detection (QoD) and communication cost through dynamic adaptation.

Index Terms—Transient signal detection, adaptive sampling, missing samples.

I. INTRODUCTION

Consider a sensor field illustrated in Fig. 1 where sensors are deployed to detect certain transient events such as gun shots, moving targets, or sudden changes in the environment. After taking measurements, the sensors report them to a fusion center through erasure channels, where some of the samples may be lost. The reporting can be done through either in-network routing or direct transmission (e.g., SENMA [1]). Missing samples can be caused by fading, interference, network congestion, etc. Because of the transience of the signal, some samples may carry more information than the others, and it is not clear how to recover the detection performance once these significant samples are lost. Applications of this problem include target detection, military surveillance, environmental monitoring, etc.

Suppose that the fusion center can control the sampling rate of the sensors through a broadcast feedback channel. Then to compensate for missing samples, a trivial strategy is to keep sampling at the maximum rate. This strategy will, however, generate a large amount of traffic, resulting in more congestion and sample loss and draining the sensors’ energy. Therefore, we need a sampling strategy which can mitigate the effect of missing samples and achieve desired detection performance without causing excessive burden to the underlying network.

A. Related Work

Due to their wide applicability and potential for a revolutionary change in monitoring and surveillance, sensor networks are an active area of research [2], [3]. Most of the research work in this area has focused on communications/networking aspects such as low power sensor design [4], energy-efficient routing [5], multiple access [6], network lifetime maximization [7], node localization [8] and self-organization [9], etc. While this body of literature is concerned with network layer issues in sensor networks, our focus is on quantifying the performance of detection based on data gathered by the sensor nodes.

The detection framework in this paper belongs to distributed detection, where the focus has been on achieving a centralized decision under communication constraints on distributed points of measurements. Prevailing frameworks model such constraints as the maximum number of quantization levels [10] or information-theoretic channel capacities [11]. In the former framework, it is known that the optimal compression and detection rules are based on likelihood ratios; in the
latter, coding schemes have been used to translate the problem into a centralized detection on auxiliary hypotheses. Besides communication constraints, related work has also addressed other challenges, e.g., spatial correlation between sensor measurements [12], and faulty sensors [13]. Our work is different from the above in that we explicitly consider the effect of missing samples at the fusion center; furthermore, unlike previous work which typically assumes stationary hypotheses, our hypotheses are nonstationary, due to which the lost samples cannot be recovered, and the detection has to be performed in their absence.

Error correction coding is a commonly used technique in wireless communication to reduce the probability of sample missing, since certain errors can be corrected with the code [14]. While such an approach can reduce the likelihood of missing a sample, it introduces additional complexity at the sensor nodes and extra coding delays at the fusion center.

B. Summary of Results

We consider the detection of transient signals in noise with incomplete observations. Joint design of the sampling and the detection strategies is considered, with the focus on adapting sampling rate to guarantee certain detection performance in the presence of missing samples.

Specifically, by analyzing the performance degradation caused by missing samples, we reduce the sampling problem to one of accumulating sufficient signal energy under given accuracy requirement. From the design perspective, we use as performance criteria not only the Quality of Detection (QoD), measured by accuracy, timeliness, and robustness, but also the communication cost measured by sampling rate to achieve a balanced design. An adaptive sampling strategy is proposed based on prior knowledge about the distribution of missing samples as well as their realizations to adjust the sampling rate as missing occurs according to given QoD requirements. Since the optimal strategy that minimizes sampling rate under QoD constraints is generally intractable, the proposed strategy offers a suboptimal alternative with low complexity and substantial improvement over fixed-rate sampling. For exponentially-decaying signals and i.i.d. model of missing samples, we obtain closed-form solutions for the proposed strategy and verify its performance through simulations.

The rest of the paper is organized as follows. Section II formulates the problem. Section III motivates and proposes a solution to adaptive sampling, which is then tested in simulations in Section IV. Section V discusses alternative system architectures, and Section VI concludes the paper with remarks.

II. THE DETECTION PROBLEM

We use the convention that uppercase letters denote random variables, lowercase realizations, boldface vectors, and plain scalars.

Consider the following detection problem. As illustrated in Fig. 2, $K$ sensors measure a signal in noise by taking samples $R_{ji}$ ($j = 1, \ldots, K$), which are then sent to a fusion center through erasure channels $Y_{ji} = H_{ji}R_{ji}$, where each $H_{ji}$ is a binary random variable with $H_{ji} = 0$ denoting erasure. The fusion center observes measurements drawn according to one of the following hypotheses:

$$
\mathcal{H}_0 : Y_{ji} = H_{ji}W_{ji}, \quad j = 1, \ldots, K, \quad i = 1, \ldots, n,
$$

$$
\mathcal{H}_1 : Y_{ji} = H_{ji}(s_{ji} + W_{ji}), \quad j = 1, \ldots, K, \quad i = 1, \ldots, (\delta)
$$

where $s_{ji}$’s are sampled signals and $W_{ji}$’s sensing noise. Suppose the signals are known, and both the noise and the erasure are independent across sensors\textsuperscript{1}. Furthermore, assume $W_{ji}$’s are i.i.d. white Gaussian noise with mean 0 and variance $\sigma^2$. Note that under the independence assumption, the multi-sensor scenario is a trivial extension of the single-sensor scenario, and therefore in the sequel we will focus on the case that $K = 1$. Moreover, the fusion center can observe which samples are missing. Here we consider a simple sensor model where most of the processing takes place at the fusion center\textsuperscript{2}. Alternative architectures are discussed later in Section V.

We use a Neyman-Pearson framework for detection. Given false alarm constraint $\alpha$ ($\alpha \in (0, 1)$), it can be shown that the Neyman-Pearson detector does not have a closed form. Therefore, we approximate it by a conditional Neyman-Pearson detector\textsuperscript{3} $(\delta(\mathbf{y}^n_{\mathbf{h}}(y_i)_{i=1}^n, \mathbf{h}(h_i)_{i=1}^n))$

$$
\delta(\mathbf{y})|_{\mathbf{h}} = \begin{cases} 
1 & \text{if } \sum_{i=1}^n s_i y_i \geq \sigma Q^{-1}(\alpha)\sqrt{e_n} \\
0 & \text{o.w.,}
\end{cases} \quad (2)
$$

where $e_n$ is the signal energy (under $\mathcal{H}_1$) in the received samples\textsuperscript{4}, defined as $e_n \overset{\Delta}{=} \sum_{i=1}^n h_i s_i^2$. It can be shown that given $\mathbf{h}$, $\delta$ maximizes the detection probability within false alarm constraint $\alpha$.

\textsuperscript{1}The sensing noise at different sensors is naturally independent, whereas independent erasure is a limiting assumption, which will be generalized to model MAC layer in future work.

\textsuperscript{2}Instantaneous coding, e.g., through repetitions, is allowed and can be incorporated into the distribution of missing samples.

\textsuperscript{3}Here $Q(\cdot)$ is the tail probability for the standard Gaussian distribution.

\textsuperscript{4}Note that the unit of $e_n$ is different from the physical unit of energy.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.png}
\caption{A detection system consisting of sensing, transmission, detection, and feedback control modules.}
\end{figure}
III. ADAPTIVE SAMPLING

Given the detection system in Section II, we hope to tune the system to achieve desired quality of detection (QoD) regardless of which samples are missing at the minimum expense of resource. In this section, we will (i) motivate adaptive sampling by analyzing the degradation of detection performance caused by missing samples, (ii) formulate a set of performance criteria to guide our design, and (iii) propose an adaptive sampling strategy with tunable parameters and analyze its performance.

A. QoD Degradation under Missing Samples

It is easy to see that missing samples negatively affect the quality of detection; how it is affected, however, requires closer examination. Given the detector $\delta$ in (2), the conditional miss probability for given $h$ is given by

$$P_M(\delta|Y|h) = 1 - Q\left(Q^{-1}(\alpha) - \frac{\sqrt{e_n}}{\sigma}\right) \geq F(e_n; \alpha, \sigma).$$  

A key observation from (3) is that missing samples affect the detection performance through the received signal energy $E_n$. For time-varying signals in which $s_i$’s may be different, the direct consequence is that the detection performance depends on not only how many samples are missing, but also which samples are missing. In particular, in the detection of transient signals, the performance is rather sensitive to the missing of initial samples because they typically contain the majority of energy.

Given the distribution of $H$ that governs sample missing, the detection performance is characterized by the overall miss probability

$$P_M(\delta|Y) = \mathbb{E}[F(E_n; \alpha, \sigma)] 
\geq F\left(\mathbb{E}\left[\sum_{i=1}^{n} H_i s_i^2\right]; \alpha, \sigma\right) \geq F\left(\sum_{i=1}^{n} (1 - q_i) s_i^2; \alpha, \sigma\right),$$

where the expectation is taken over $H$, and $q_i$ is the (marginal) probability of missing the $i$th sample. The inequality (4) is because $F(x; \alpha, \sigma)$ is (almost) convex in $x$. Note that this bound holds even if $H_i$’s are correlated.

B. Performance Criteria

Having characterized the performance degradation caused by missing samples, we now proceed to seek for countermeasures. Unlike constant signals where samples tend to be treated equally, samples from transient signals bear different significance depending on when they are taken, and a significant sample may not be replaced by a less significant one taken later. A straightforward approach to compensate for the loss is to increase the sampling rate so that there will be more future samples, each with less signal energy, to achieve the same detection performance. This is the basic idea of adaptive sampling. To guide our design, we first need to specify how the detection system will be evaluated. There are multiple desirable properties of the detection system, which we partition into two sets, one for QoD and one for resource consumption.

For QoD, we use the accuracy, timeliness, and robustness of detection as performance measure. For accuracy, a commonly used measure is the error probabilities. For timeliness, a natural measure is the detection delay, defined as the elapsed time before a decision is made. It remains to define a measure for robustness. Conceptually, we want to measure to what extent the sampling strategy can recover the loss of missing samples. Ideally, for continuous signals, we can always recover the lost energy if the sampling rate can be increased unboundedly. In practice, however, the sampling device usually imposes a maximum sampling rate, or a minimum sampling period $T_{\text{min}} > 0$. If $T_{\text{min}} \geq \epsilon > 0$, then the desired accuracy cannot be guaranteed for sure (e.g., the first $n$ samples are missing for $n$ sufficiently large). Thus, the probability of achieving given accuracy effectively measures the robustness of the sampling strategy against sample missing. Alternatively, one may use the absolute number of tolerable missing samples under the desired accuracy as a measure of robustness. We, however, point out that such a criterion will drive the detector to delay the decision so as to accumulate more missing samples, contradicting the timeliness requirement, and is thus not adopted in our framework.

For resource consumption, the main concerns are communication costs for sensors, including both bandwidth consumption (measured by the average sampling rate) and total load (measured by the number of samples, i.e., sample size). Note that since the signal is decaying over time, minimizing sample size has the same effect as maximizing robustness and minimizing delay, both resulting in a trivial solution of using a constant sampling period $T_{\text{min}}$. Minimizing sampling rate, on the other hand, requires us to use as large sampling periods as possible, leading to a tradeoff between detection performance and resource consumption. The tradeoff among various performance criteria is illustrated in Fig. 3.

![Fig. 3. Opposite influences of various performance criteria on the selection of sampling period $T$.](image)

C. Adaptive Sampling Strategy

Given the performance criteria, an immediate question is how to adapt sampling rates to achieve desired optimization. To this end, we introduce the following notions to model the variables in the system: given required accuracy $(\alpha, \beta)$, as illustrated in Fig. 4.
1) compute the residual energy required to achieve \((\alpha, \beta)\) by 
\[
\Delta E_i = \Delta E_{i-1} - E_i (i \geq 1) \quad \text{and} \quad \Delta E_0 = F^{-1}(\beta, \alpha, \sigma);
\]
2) compute the new sampling period \(T_i\) by 
\[
T_i = \max(T_{\text{min}}, T_i),
\]
where \(T_i\) is such that using fixed sampling period \(T_i\) starting from stage \(i\) will achieve given accuracy with probability \(\gamma\), i.e., 
\[
\Pr\left\{ \sum_{j=1}^{(d-t_i)/\tilde{T}_i} s^2(t_i + j\tilde{T}_i)h_j \geq \Delta E_i \right\} = \gamma. \tag{5}
\]

The sampling continues until detection completes or time \(d\) is reached. Note that since the \((i-1)\)th stage completes at the \(i\)th missing sample, realizations of \(E_{i-1}\) and \(t_i\) are known.

The above strategy reflects the idea of QoD-driven design, where we directly start from the desired QoD (measured by error probabilities, detection delay, and robustness) and tune the detection system to achieve this QoD with the minimum cost (measured by sampling rate). The merit of this design is that it is easy to configure because the key QoD measures are modeled as explicit parameters. Moreover, it is also easy to provide performance guarantee. We say that the QoD \((\alpha, \beta, d, \gamma)\) is achievable if there exists a detection system with error probabilities no larger than \((\alpha, \beta)\), delay within \(d\), and robustness at least \(\gamma\). For sufficiently small \(T_{\text{min}}\), we have the following results.

\[
\text{Proposition 3.1:} \quad \text{As } T_{\text{min}} \to 0, \text{ the proposed adaptive sampling strategy combined with the detector in (2) can achieve any achievable QoD specified by } (\alpha, \beta, d, \gamma).
\]

**Proof:** The delay constraint is easily verified. As \(T_{\text{min}} \to 0\), the maximum sampling rate constraint is removed, and thus the proposed sampling strategy is guaranteed to collect sufficient energy to achieve accuracy \((\alpha, \beta)\) with probability \(\gamma\) even if there is no more adaptation. Further adaptation will only increase the robustness, and hence the results hold. 

In addition to satisfying the QoD requirement, the proposed strategy also aims at reducing communication cost by minimizing sampling rate. Although this strategy is in general not optimal at minimizing sampling rate, it is simple to use and significantly improves the fixed-rate sampling as shown later in Section IV.

The proposed adaptive sampling strategy is not tied to any specific types of signals or sample missing models. Given a signal and a model, we need to implement step (2) based on the distribution of the signal energy to be received, i.e., 
\[
\sum_{j=1}^{(d-t_i)/\tilde{T}_i} s^2(t_i + j\tilde{T}_i)h_j.
\]
Since this distribution often cannot be computed in closed form, approximations are needed. For example, for an exponentially-decaying signal \(s^2(t) = e^{-\lambda t}\) \((\lambda > 0)\), and a model where missing samples occur i.i.d. with probability \(q\), we can replace the decaying signal with a constant signal of the same mean and reduce (5) to 
\[
\Pr\left\{ \frac{\tilde{T}_i e^{-\lambda(t_i + t_i)} (1 - e^{-\lambda(d-t_i)})^{(d-t_i)/\tilde{T}_i} \sum_{j=1}^{(d-t_i)/\tilde{T}_i} h_j}{(1 - e^{-\lambda \tilde{T}_i})(d-t_i)} \geq \Delta E_i \right\} = \gamma.
\]
Now that \( \sum_{j=1}^{\lfloor (d-t_i)/\hat{T}_i \rfloor} h_j \) can be approximated as a Gaussian random variable, it can be shown that \( \hat{T}_i \) has a closed-form solution

\[
\hat{T}_i = \frac{(c + \sqrt{c^2 + 4\lambda t_i}))}{4\lambda a^2},
\]

where \( a = ((d-t_i)e^{\lambda t_i}E_3(1-e^{-\lambda(d-t_i)})) \), \( b = (d-t_i)(1-q) \), and \( c = Q^{-1}(\gamma)\sqrt{(d-t_i)(1-q)} \).

IV. SIMULATIONS

In this section, we simulate the proposed adaptive sampling strategy to verify its performance. The main questions of interest are: (a) how the performance is affected by environmental factors such as the noise level, the minimum sampling period, and the probability of missing samples, and (b) how the proposed adaptive sampling strategy compares with fixed-rate sampling. We use an exponentially-decaying signal \( s^t(t) = e^{-\lambda t} \) and i.i.d. model for missing samples with missing probability \( q \).

We first plot sampling rate, detection delay, and robustness as functions of the noise standard deviation \( \sigma \) under several constraints on the minimum sampling period \( T_{\text{min}} \); see Fig. 5–7. As noise increases, the sampling rate increases and the robustness decreases monotonically as expected. The delay curves (Fig. 6), however, are not monotone with noise. They remain constant (with slight increase) up to certain noise levels, after which delays drop suddenly. This is because due to \( T_{\text{min}} \), the system has a limit on tolerable noise level (for given accuracy); within this limit, the system can adapt, but beyond this limit, the system will saturate and the detection will fail. The saturation can also be observed in the other plots (Fig. 5, 7), where after certain point the rate stops increasing and the robustness decreases sharply. Moreover, the maximum tolerable noise level decreases as \( T_{\text{min}} \) increases and the system becomes more constrained. We have similar observations when varying the missing probability \( q \) (plots omitted due to space limit).

Next, we compare the proposed adaptive sampling with fixed-rate sampling through the delay-rate and robustness-rate tradeoffs; see Fig. 8 and 9. The plots show that adaptive sampling outperforms fixed sampling in that it can achieve much better robustness and smaller delay at the same average sampling rate. Moreover, it can be configured to operate at various tradeoff points by having different design parameters, providing greater flexibility. As \( \gamma \) increases, the sampling rate increases, the detection delay decreases, and the robustness increases. As \( d \) increases, however, the robustness increases significantly, but the delay is similar. This is because \( d \) is used as a strict deadline beyond which detection is aborted. These observations suggest that we use large \( d \) and tailor \( \gamma \) to achieve desired robustness and delay within tolerable sampling rate.

V. DISCUSSIONS

In this paper, we have assumed that sensors do not process the samples (except possible instantaneous coding before transmission), and all the processing and control occur at the fusion center. We point out that this is a limiting assumption, and the performance can be improved by enriching the sensors’ capability. In particular, if sensors can code across samples, then they can just send the sufficient statistics (which is \( \sum_{i=1}^{n} s_i \) for AWGN) to the fusion center, and any missing sample will be (optimally) incorporated in future samples. On the one hand, this approach eliminates the need of adaptive sampling unless it is crucial to control detection delay; on the other hand, it imposes additional complexity at the sensors. More importantly, such an approach limits the detection system to a specific signal, the one used to compute sufficient statistics by the sensors, and therefore does not scale if we need to test against a large number of signals because the sensors have to generate one sufficient statistic for each signal. For the relative performance of this approach as compared with the direct transmission approach, we refer to [15].

VI. CONCLUSION

This paper addresses how to adapt the sampling process to compensate for missing samples while conserving network.
Performance of adaptive sampling under varying noise level $\sigma$

$(\lambda = 1, \alpha = \beta = 0.05, q = 0.3, 10^5$ Monte Carlo runs. Design parameters: $d = 2, \gamma = 0.7$.)

Resource for detecting transient signals in noise. The focus is on the tradeoff between QoD and communication cost. An adaptive sampling strategy is proposed, which combined with the optimal detector can achieve given QoD requirements at reduced communication cost. Although throughout the paper, adaptive sampling in temporal domain is considered, the techniques are also applicable to spatial domain (e.g. sampling by a mobile access point), where the signal decays over distance and the causality constraint translates into a one-directional movement of the sampling agent. Since in many transient events, signal strength decays both over time and over distance, the proposed method can be applied in both dimensions to achieve better performance.

REFERENCES