Multi-Sensor Centralized Tracking System

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Chapter 1

Introduction

Kinematic state refers to a vehicle’s position, velocity, and attitude. The kinematic state of a vehicle can be defined by specifying the relative position, velocity, and orientation of two reference frames. The two reference frames typically used are a vehicle fixed body frame and a navigation frame with known orientation. In this application, we consider the vehicle’s position and velocity as the kinematic state and we select a North-East-Down (NED) frame as the navigation frame. A tracking system is used to estimate the kinematic state of the vehicle or, more specifically, to estimate the position vector and velocity vector of the body frame relative to the navigation frame.

In general, a tracking system consists of a set of sensors to measure the target vehicle’s kinematic state; a stochastic system generated from a dynamic model, measurement model, and sensor measurement models; and a filter that blends the sensor measurements to compute estimates of the target vehicle’s kinematic state. The dynamic model describes the time evolution of the target vehicle’s kinematic state and incorporates uncertainty of the vehicle’s motion. The measurement model relates the sensor measurements to the vehicle’s kinematic state. The sensor measurements models describe the sensor errors and performance characteristics, and the models can be incorporated into either the dynamic or measurement models. In this application, we assume that the target vehicle is traveling in the N-E plane of an NED frame and we design a multi-sensor centralized tracking (MSCT) system to track the target vehicle.

A MSCT system consists of multiple sensors located within a predefined tracking zone, a sensor network, a predefined central processing location (CPL), and the remaining components of general tracking systems listed above. In a MSCT system, a sensor takes measurements of the target vehicle at a specified time. Each sensor is located at a node which can transmit and receive data from adjacent nodes. The sensor measurements are then transmitted through the nodes, or sensor network, to the central location for processing. The CPL has the necessary hardware to transmit and receive data from adjacent sensor nodes, process the sensor measurements, run the filter, and track the target vehicle.

There are a number of timing and measurement assumptions and errors that are inherent to a MSCT system. We make the following assumptions regarding the MSCT system.

- The measurement times for each sensor within the sensor network are synchronized.
- The sensor measurements are part of a sensor data packet that includes the measurements, the sensor ID, and the measurement time stamp.
- The sensor nodes and the CPL can communicate only with adjacent nodes.
- The time period required for a sensor data packet transmitted from a sensor node to an adjacent node or the (adjacent) CPL is fixed. We will refer to this time period as the transmission time.
- The sensor nodes and CPL transmit or receive a sensor data packet without a time delay. (This time delay refers only to the local process of transmitting or receiving a sensor data packet.)

The timing errors of the MSCT system are
- sensor measurement time synchronization errors
- sensor data packet time-of-arrival delays at the CPL
- asynchronous, or out-of-sequence, sensor data packets arrivals to the CPL

Ideally, all sensors in the sensor network take measurements of the target vehicle at the same time. Sensor measurement time synchronization errors refer to the time difference between the actual measurement times of each sensor and the desired time. We will assume that these errors are much shorter than the time between consecutive sensor measurements and, thus, ignore these errors.

The sensor nodes transmit the sensor data packets to the CPL through adjacent nodes in the sensor network. Depending on the sensor node locations, a node may have either one adjacent node or multiple adjacent nodes. Furthermore, each node has limited communication bandwidth and, thus, there may be a queue of sensor data packets waiting to be transmitted from a node. In this case, if there is a second adjacent node with a shorter queue of sensor data packets waiting to be transmitted, then the sensor data packet may be transmitted to this adjacent node. Therefore, the transmission path from a sensor node to the CPL through the sensor network is not necessarily fixed. However, we will assume that the transmission time from a sensor node to the CPL is bounded.

The sensor network itself leads directly to two types of timing errors. First, there is a delay between the sensor measurement time and the time-of-arrival of the sensor data packet at the CPL because each node can only transmit a sensor data packet to an adjacent node. Second, the CPL may receive sensor data packets from a particular sensor node out-of-sequence because the transmission path from the node to the CPL is not fixed. In other words, sensor data packets with a more recent time stamp may arrive at the CPL before sensor data packets with older time stamps.

The measurement errors of the MSCT system are

- sensor node position uncertainty
- sensor measurement wide band noise
- sensor performance errors due to sensor characteristics

Sensor node position uncertainty refers to localization errors of each node. Sensor wide band noise and performance errors depend on the type of sensor used to track the target vehicle. These errors are included in the sensor measurement models.

### 1.1 Objectives

There are three estimation objectives for the MSTC system and its various timing errors.

- show that the estimated state covariance matrix is bounded
- show that the bias errors of the estimated state mean vector (target vehicle’s kinematic state) are bounded
- determine the effect of various timing and measurement errors on the bias error of the estimated state mean vector

The estimated state covariance matrix can be considered as a statistical description of the estimation errors of the state mean vector and can be used as a metric to assess whether these estimation errors have bounded variance. The bias errors of the estimated state mean vector are simply the difference between the estimated and true state mean vectors. If the bias errors of the estimated state mean vector are bounded, then the estimated state mean vector converges, within the bound, to the true state mean vector. A bounded state covariance matrix is a necessary condition for a bounded bias error of the estimated state mean vector.

In this paper, we perform the following tasks. First, we design a stochastic system to model the motion of the target vehicle and to incorporate the following sensor measurement errors.
• sensor node position uncertainty
• sensor measurement wide band noise
• sensor performance errors due to sensor characteristics

Second, we design a Kalman filter to blend sensor measurements from the sensor network to compute estimates of the target vehicle’s kinematic state in real-time. Furthermore, we design the Kalman filter to incorporate the following timing errors.

• sensor data packet time-of-arrival delays at the CPL
• asynchronous, or out-of-sequence, sensor data packets arrivals to the CPL

We will refer to this filter as the tracking filter throughout this paper.

This tracking filter modifies and extends the design of the hybrid-tracking filter designed in [ASYCS].

First, we assume that all sensors within the sensor network measure the relative position of the target vehicle. Second, the dynamic model of the stochastic system is modified to ensure the system is observable. The observability of the stochastic system is a necessary condition for the filter to computed bounded, minimum variance estimates of the state mean vector (target vehicle’s kinematic state). Third, we incorporate the timing errors of the sensor network into the filter design. These modifications allow us to study the effect of varying parameters of the sensor network on the estimation errors of the target vehicle’s kinematic state.

1.2 Organization

This paper is organized as follows. In Chapter 2, we provide a brief summary of the Kalman filter. In Chapter 3, we design the stochastic system for the MSCT system. In Chapter 4, we design the Kalman filter for the MSCT system. In Chapter 5, we evaluate the performance of the tracking filter designed in Chapter 4.
Chapter 2

The Kalman Filter

In certain applications, the performance objectives of a filter are to compute unbiased, minimum variance estimates of a state mean vector from a set of measurements corrupted by noise. These performance objectives influence the selection of physical components such as sensors, their performance characteristics, and their locations. Furthermore, these performance objectives also influence the selection of sensor measurement models, uncertainty of the dynamic model, the statistics of the initial state vector, and, possibly, the state vector trajectory.

The Kalman filter (KF) is used to estimate the statistics of a state vector whose time evolution is governed by a stochastic system using a two step prediction-correction procedure. The KF, under various assumptions on the system, can be considered as a model based algorithm that is used to recursively estimate both the state mean vector and state covariance matrix. The optimality of the KF in a state vector estimation problem depends on the linearity of the stochastic system, the statistics of the initial state vector, and the statistics of the noise vectors. If the stochastic system is linear, then the KF is the optimal mean squared error estimator. If the stochastic system is nonlinear, then an extension to the KF, the extended Kalman filter (EKF), is a standard sub-optimal algorithm used to estimate the statistics of the state vector.

There are many variations of the EKF, however, the underlying approach of the EKF is to approximate the nonlinear system models by linearizing these models and to apply the linear measurement update equations in combination with the linearized system models. In the prediction step, the state mean vector is propagated using the dynamic model regardless of its classification as linear or nonlinear. The state covariance matrix is propagated using the dynamic model linearized about the most recent estimate of the state mean vector. In the correction step, the predictions of the state mean vector and state covariance matrix are updated using the linear measurement update equations and the measurement model linearized about the most recent predicted estimate of the state mean vector.

In this chapter, we provide an overview of the EKF. In Section 2.1, we describe the general nonlinear estimation problem. In Section 2.2, we describe the assumptions made on a stochastic system for implementation of the EKF. In Section 2.3, we describe the two step estimation algorithm and further assumptions made on the stochastic system for implementation of the EKF. In Section 2.4, we state the Riccati equation that governs the time evolution of the estimated state covariance matrix. In Section 2.5, we summarize the discussion of the EKF. We note that Section 2 is extracted from [BAG] with key references [AM], [CJ], [SIM], and [STE].

2.1 Nonlinear State Vector Estimation: A Bayesian Approach

The objective of a state vector estimation problem is to estimate the statistics of a state vector whose time evolution is governed by a stochastic system using measurements corrupted by noise. We consider discrete nonlinear stochastic systems of the form

\[ x_{k+1} = f_k(x_k, w_k) \]
\[ z_k = h_k(x_k, v_k) \]  \hspace{1cm} (2.1)
where at time $t_k$, $x_k$ refers to the $n \times 1$ state vector, $w_k$ refers to the $m \times 1$ process noise vector, $z_k$ refers to the $p \times 1$ measurement vector, $v_k$ refers to the $p \times 1$ measurement noise vector, $f_k$ refers to the dynamic model, and $h_k$ refers to the measurement model. We denote $Z_k$ as the sequence of measurement vectors available at time $t_k$ where $Z_k = [z_0, \ldots, z_k]$.

A Bayesian estimator can be considered as an algorithm that is used to estimate the statistics of a state vector or, more specifically, to estimate the probability density function (pdf) of the posterior state vector, $p(x_k, Z_k)$, at time $t_k$. This estimator computes estimates of the statistics of a state vector using a two step procedure at times $t_k$ and $t_{k+1}$.

In the first step, the statistics of the state vector at time $t_{k+1}$ are predicted using the statistics of the state vector at time $t_k$ and the measurement sequence $Z_k$. The statistics of the predicted state vector, as specified in equation 2.1, can be computed using the integral

$$p(x_{k+1}|Z_k) = \int p(x_{k+1}|x_k, Z_k)p(x_k|Z_k)dx_k$$

where $p(x_k|Z_k)$ refers to the pdf of the posterior state vector at time $t_k$.

In the second step, the statistics of the predicted state vector, $p(x_{k+1}|x_k, Z_k)$, are corrected using the current measurement vector, $z_{k+1}$, to yield the statistics of the posterior state vector at time $t_{k+1}$. The statistics of the posterior state vector can be computed using Bayes’s formula

$$p(x_{k+1}|Z_{k+1}) = \frac{p(z_{k+1}|x_{k+1})p(x_{k+1}|Z_k)}{p(z_{k+1}|Z_k)}$$

where $p(z_{k+1}|x_{k+1})$ refers to the pdf of the measurement vector given the current state vector, as specified in equation 2.1, and $p(z_{k+1}|Z_k)$ is a normalization constant

$$p(z_{k+1}|Z_k) = \int p(x_{k+1}|Z_k)p(z_{k+1}|x_{k+1})dx_k$$

Estimating the statistics of the posterior state vector requires computation of the pdf of the predicted state vector, $p(x_{k+1}|Z_k)$. The integral expression for $p(x_{k+1}|Z_k)$ does not have a closed form solution for arbitrary multivariate pdfs. Therefore, these integrals are solved using numerical approximations based on a set of assumptions. The Kalman filter is a class of Bayesian estimators that uses assumptions on the stochastic system so that the integral expressions for $p(x_{k+1}|Z_k)$ can be computed exactly. These assumptions include the statistics of the process noise vector, measurement noise vector, and initial state vector; and the form of the dynamic and measurement models of the stochastic system.

### 2.2 Stochastic System

The Kalman filtering approach to estimate the statistics of a state vector governed by equation 2.1 is based on two initial assumptions regarding the statistics of the stochastic system.

- The noise vectors are additive, Gaussian sequences.
- The state vector is a Markov sequence with a Gaussian distribution.

These assumptions simplify the integral expressions for $p(x_{k+1}|Z_k)$. Assumption 1 implies that $p(x_k|Z_k)$ can be approximated as a multivariate Gaussian distribution. Assumption 2 implies that $p(x_{k+1}|Z_k)$ depends only on the statistics of the state vector at time $t_k$ and can be computed without explicit reference to the statistics of the state vector from times $t_0$ to $t_{k-1}$. Furthermore, these assumptions imply that the statistics of the posterior state vector can be entirely characterized by the first moment and second central moment of $p(x_{k+1}|Z_{k+1})$. We note that second central moment of a random vector equals the second moment of a random vector with a deterministic adjustment using the mean vector of the random vector.

Based on these two assumptions, we modify equation 2.1 and consider discrete nonlinear stochastic systems of the form

$$x_{k+1} = f_k(x_k) + \Gamma_k w_k$$
$$z_k = h_k(x_k) + v_k$$

(2.5)
where at time $t_k$, $\Gamma_k$ refers to the $n \times m$ process noise mapping matrix. The vectors $w_k$ and $v_k$ are modeled as zero-mean, Gaussian, uncorrelated white sequences with

\[
E \{ w_k w_k^T \} = Q_k, \quad Q_k \geq 0 \\
E \{ v_k v_k^T \} = R_k, \quad R_k > 0 \\
E \{ w_i v_j^T \} = 0, \quad \forall i, j
\]

where at time $t_k$, $Q_k$ refers to the $m \times m$ state process noise covariance matrix and $R_k$ refers to the $p \times p$ measurement noise covariance matrix. We make the following assumptions regarding the statistics of the stochastic system.

- The models $f_k$ and $h_k$ are $C^1$ functions.
- The pdf of the initial state vector is Gaussian with mean vector $\bar{x}_0$ and covariance matrix $P_0$

$$E \{ (x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T \} = P_0 \quad (2.6)$$

- The vectors $w_k$ and $v_k$ are uncorrelated with $x_0$

$$E \{ (x_0 - \bar{x}_0)w_i^T \} = 0, \forall i = 1, \ldots, m$$
$$E \{ (x_0 - \bar{x}_0)v_j^T \} = 0, \forall j = 1, \ldots, p$$

### 2.3 Two Step Estimation Algorithm

The EKF is used to estimate the statistics of the state vector governed by nonlinear stochastic systems as specified in equation 2.5 using the following two step procedure at times $t_k$ and $t_{k+1}$.

#### 2.3.1 Prediction Step

At time $t_k$, the state mean vector and state covariance matrix are predicted using the measurement sequence $Z_k$

\[
\dot{x}_{k+1|k} = E \{ f_k(x_k) + \Gamma_k w_k | Z_k \} = E \{ f_k(x_k) | Z_k \} \sim f_k (E \{ x_{k|k} \}) = f_k (\hat{x}_{k|k}) \quad (2.7)
\]

\[
P_{k+1|k} = E \{ \delta x_{k+1|k} \delta x_{k+1|k}^T | Z_k \} \sim \Phi_k P_{k|k} \Phi_k^T + \Gamma_k Q_k \Gamma_k^T \quad (2.10)
\]

where $\delta x_{k+1|k}$ refers to the predicted state error vector

$$\delta x_{k+1|k} = x_{k+1} - \hat{x}_{k+1|k} \quad (2.11)$$

We note that the equation which forms the basis for equation 2.10 be derived by writing the dynamic model equation 2.5 using a Taylor series expansion about $\hat{x}_{k|k}$

\[
x_{k+1} = (f_k(x) + \Gamma_k w_k) |_{\hat{x}_{k|k}} + \frac{\partial (f_k(x) + \Gamma_k w_k)}{\partial x} |_{\hat{x}_{k|k}} \delta x_{k|k} + O \left( \delta x_{k|k}^2 \right) \quad (2.12)
\]

\[
O \left( \delta x_{k|k}^2 \right) = \sum_{i=2}^{\infty} \frac{1}{i!} (\delta x_{k|k} \cdot \nabla_x)^i (f_k(x) + \Gamma_k w_k) |_{\hat{x}_{k|k}} \delta x_{k|k} \quad (2.13)
\]
If equations 2.12 and 2.8 are substituted into equation 2.11, then $\delta x_{k+1|k}$ can be written as

$$
\delta x_{k+1|k} = f_k (\hat{x}_{k|k}) + \Gamma_k w_k + \frac{\partial f_k(x)}{\partial x} \delta x_{k|k} + O \left( \delta x^2_{k|k} \right) - f_k (\hat{x}_{k|k})
$$

$$
\delta x_{k+1|k} = \Phi_{k} \delta x_{k|k} + \Gamma_k w_k
$$

(2.14)

where at time $t_k$, $\Phi_k$ refers to the Jacobian of the dynamic model

$$
\Phi_k = \left. \frac{\partial f_k(x)}{\partial x} \right|_{\hat{x}_{k|k}}
$$

(2.15)

and $O \left( \delta x^2_{k|k} \right) \sim 0$.

Equation 2.8 suggests that the estimated state mean vector is propagated using the dynamic model regardless of its classification as linear or nonlinear. However, if the dynamic model is nonlinear, then the estimated state covariance matrix is propagated using the Jacobian of the dynamic model. This prediction step, or time update, requires the following assumptions for implementation.

- $O \left( \delta x^2_{k|k} \right) \sim 0$.
- The pdf of the state vector is Gaussian at every time step following linearization.
- The a priori state mean vector at time $t_{k+1}$ can be predicted by propagating the posterior state mean vector at time $t_k$ using the dynamic model.
- The state mean vector and state covariance matrix can be predicted using separate equations.

### 2.3.2 Correction Step

At time $t_{k+1}$, the predicted state mean vector and predicted state covariance matrix are corrected using the measurement vector $z_{k+1}$ and the linear measurement update equations

$$
\hat{z}_{k+1|k} = E \left\{ z_{k+1|k} | x_k, Z_k \right\}
$$

$$
\sim \tilde{z}_{k+1} = E \left\{ x_{k+1|k} \right\}
$$

(2.16)

$$
\nu_{k+1} = z_{k+1} - \hat{z}_{k+1|k}
$$

(2.17)

$$
\nu_{k+1}^T = P_{zz,k+1}^{-1} P_{uv,k+1}^{-1}
$$

(2.18)

$$
K_{k+1} = P_{k+1|k} H_{k+1}^T \left( R_{k+1} + H_{k+1} K_{k+1} H_{k+1}^T \right)^{-1}
$$

(2.19)

$$
\tilde{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \nu_{k+1}
$$

(2.20)

$$
P_{k+1|k+1} = P_{k+1|k} - K_{k+1} P_{uv,k+1} K_{k+1}^T
$$

(2.21)

$$
P_{k+1|k+1} = P_{k+1|k} - H_{k+1} K_{k+1} P_{k+1|k}
$$

(2.22)

where at time $t_{k+1}$, $\nu_{k+1}$ refers to the $p \times 1$ innovation vector, $\hat{z}_{k+1|k}$ refers to the $p \times 1$ predicted measurement vector, $K_{k+1}$ refers to the $n \times p$ Kalman gain matrix, $P_{uv,k+1}$ refers to the innovation covariance matrix, and $P_{zz,k+1}$ refers to the cross-correlation matrix between the predicted state mean vector and predicted mean measurement vector. If the measurement model is nonlinear, then the correction step is performed using the Jacobian of the measurement model where

$$
H_{k+1} = \left. \frac{\partial h_{k+1}(x)}{\partial x} \right|_{\hat{x}_{k+1|k}}
$$

(2.24)

The correction step, or measurement update, requires the following assumptions for implementation.
The matrix \( P_z = R_{k+1} + H_{k+1}P_{k+1|k}H_{k+1}^T \) is nonsingular.

The matrix \( P_{k+1|k} \geq 0 \).

The matrix \( R_{k+1} > 0 \).

We note that the computation of \( \hat{z}_{k+1|k} \) can be classified as a prediction step, however, it is only performed during a measurement update.

### 2.3.3 Additional Assumption on Stochastic System

In addition to the assumptions previously stated, we make the following assumptions for all times \( t_k \).

- The pair \((\Phi_k, \Gamma_k \sqrt{Q_k})\) is controllable where \( Q_k = \sqrt{Q_k} \sqrt{Q_k}^T \).
- The pair \((\Phi_k, H_k)\) is observable.
- The five system matrices \( \Phi_k, \Gamma_k, Q_k, H_k, \) and \( R_k \) are constant throughout the time interval \([t_k, t_{k+1}]\).

### 2.4 Riccati Equation

The covariance equations of the time and measurement updates can be combined to formulate a discrete time-varying Riccati equation. If equation 2.23 is substituted into equation 2.10, then \( P_{k+1|k} \) can be written as

\[
P_{k+1|k} = \Phi_{k+l,k} P_{k|k} \Phi_{k+l,k}^T + \hat{Q}_k - \Phi_{k+l,k} K_k H_k P_{k|k-1} \Phi_{k+l,k}^T
\]  

(2.25)

This Riccati equation relates \( P_{k|k-1} \) to \( P_{k+1|k} \) without requiring the computation of \( P_{k|k} \). If a measurement update occurs every \( l \) time updates, then equations 2.23 and 2.10 can be rewritten as

\[
P_{k+l|k} = \Phi_{k+l,k} P_{k|k} \Phi_{k+l,k}^T + \hat{Q}_k - \Phi_{k+l,k} K_k H_k P_{k|k-1} \Phi_{k+l,k}^T
\]  

(2.26)

where

\[
\hat{Q}_k = \sum_{i=k}^{k+l-1} \Phi_{k+l-1,i+1} \Gamma_i Q_i \Gamma_i^T \Phi_{k+l-1,i+1}^T
\]  

(2.27)

\( \Phi_{k+l,k} = \Phi_{k+l} \cdots \Phi_k \), and \( \Phi_{k+l,k}^T = \Phi_k^T \cdots \Phi_{k+l}^T \). The Riccati equation given in equation 2.25, or more generally equation 2.26, governs the time evolution of the estimated state covariance matrix.

### 2.5 Summary

In summary, the Kalman filter, under various assumptions on the stochastic system, can be considered as a model based algorithm that is used to estimate both the state mean vector and state covariance matrix. The EKF uses linearization in combination with the linear update equations of the KF to recursively estimate the statistics of the state vector. The following list summarizes the major assumptions that are made on the stochastic system when implementing the EKF.

- The vectors \( w_k \) and \( v_k \) are additive, zero-mean, Gaussian, white noise sequences.
- The state vector is a Markov sequence with a Gaussian distribution.
- The noise vectors, \( w_k \) and \( v_k \), and the initial state mean vector \( x_0 \) are uncorrelated.
- The models \( f_k \) and \( h_k \) are \( C^1 \) functions.
- \( O(\delta x_{k|k}^2) \sim 0 \) so that nonlinear models can be approximated by first order linear approximations of the underlying Taylor series expansions of the nonlinear models.
The pdf of the state vector is Gaussian at every time step following linearization.

The a priori state mean vector at time $t_{k+1}$ can be predicted by propagating the posterior state mean vector at time $t_k$ using the dynamic model.

The state mean vector and state covariance matrix can be predicted using separate equations.

The pair $(\Phi_k, \Gamma_k \sqrt{Q_k})$ is controllable where $Q_k = \sqrt{Q_k} \sqrt{Q_k}^T$.

The pair $(\Phi_k, H_k)$ is observable.

The matrix $P_z = R_{k+1} + H_{k+1} P_{k+1|k} H_{k+1}^T$ is nonsingular.

The matrix $P_{k+1|k} \geq 0$.

The matrix $R_{k+1} > 0$.

If any of these assumptions are violated, then the estimates of the statistics of the state vector could be biased or the variance could become unbounded. Therefore, the EKF is considered a sub-optimal nonlinear estimation algorithm.

2.5.1 Summary: Algorithm

There is no one practical algorithm that can be identified as the definitive EKF. However, the following algorithm includes the basic elements characteristic of all EKFs.

Initialize the filter

$$\hat{x}_{0|0} = E\{x_0\}$$

$$P_{0|0} = E\{(x_0 - \hat{x}_{0|0})(x_0 - \hat{x}_{0|0})^T\}$$

Time Update

$$\hat{x}_{k+1|k} = f_k(\hat{x}_{k|k})$$

$$P_{k+1|k} = \Phi_k P_{k|k} \Phi_k^T + \Gamma_k Q_k \Gamma_k^T$$

where at time $t_k$, $\Phi_k$ refers to the Jacobian of the dynamic model

$$\Phi_k = \frac{\partial f_k(x)}{\partial x} \bigg|_{\hat{x}_{k|k}}$$

Measurement Update

$$\hat{z}_{k+1|k} = h_{k+1}(\hat{x}_{k+1|k})$$

$$\nu_{k+1} = z_{k+1} - \hat{z}_{k+1|k}$$

$$K_{k+1} = P_{k+1|k} H_{k+1}^T (R_{k+1} + H_{k+1} P_{k+1|k} H_{k+1}^T)^{-1}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \nu_{k+1}$$

$$P_{k+1|k+1} = P_{k+1|k} - H_{k+1} K_{k+1} P_{k+1|k}$$

where at time $t_{k+1}$, $H_{k+1}$ refers to the Jacobian of the measurement model

$$H_{k+1} = \frac{\partial h_{k+1}(x)}{\partial x} \bigg|_{\hat{x}_{k+1|k}}$$
Chapter 3

Stochastic System

In this chapter, we design the stochastic system for the tracking filter. In Section 3.1, we describe the configuration of the sensor network. In Section 3.2, we describe the dynamic model of the target vehicle. In Section 3.3, we describe the measurement model for each sensor of the sensor network.

3.1 Configuration of the Sensor Network

The configuration of the MSCT system is shown in Figure 3.1. We make the following assumptions about the sensors and CPL.

- There are a total of $N$ sensors.
- The sensors are omni-directional (360° field of view).
- The sensors are all located in the N-E plane of the NED frame.
- The location of sensor $i$ is fixed for all $i = 1, \ldots, N$.
- The location of the CPL is fixed.

The CPL is located at the origin of the NED frame. Sensor $i$ is located at position $(p_{si,E}, p_{si,N})$ relative to the location of the CPL. The target vehicle is located at position $(p_E, p_N)$ relative to the location of the CPL. The velocity of the target vehicle is $(v_E, v_N)$ relative to the CPL. The target vehicle is located at position $(p_{Ti,E}, p_{Ti,N})$ relative to the location of sensor $i$.

![MSCT System Variable Definition](image-url)
3.2 Dynamic Model

The continuous-time dynamic model for the target vehicle is based on Newton’s second law

\[
\dot{\bar{v}}(t) = \frac{\bar{F}(t)}{m}
\]

(3.1)

where \( \bar{F} \) refers to the force acting on the tracked vehicle, \( \bar{v} \) refers to the velocity of the target vehicle, and \( m \) refers to the mass of the target vehicle. We assume that the scalar components of all vectors are resolved in the NED frame.

In one-dimension, the continuous-time dynamic model of the target vehicle can be written in state-space form as

\[
\begin{bmatrix}
\dot{p}(t) \\
\dot{v}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
p(t) \\
v(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\omega_v(t)
\]

(3.2)

where \( \omega_v \) is assumed to be a zero-mean, Gaussian, white noise scalar

\[
E\{\omega_v(t)\} = 0
\]

(3.3)

\[
E\{\omega_v(t)\omega_v^T(t)\} = Q_W \delta(t - \tau)
\]

(3.4)

and \( Q_W \) refers to the power spectral density matrix of \( \omega_v \). In this application, \( \omega_v \) is the specific force acting on the target vehicle as specified in equation 3.1. For notational convenience, we define

\[
A =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\]

(3.5)

\[
\Gamma_c =
\begin{bmatrix}
0 & 1
\end{bmatrix}
\]

(3.6)

In general, equation 3.2 can be considered as a second-order kinematic model driven by a white noise acceleration vector with power spectral density \( Q_w \). The power spectral density \( Q_w \) has units of \([\text{distance}]^2/\text{time}^3\). A small value of \( Q_w \) indicates that the target vehicle is moving with a nearly constant velocity vector. A large value of \( Q_w \) indicates that the target vehicle is accelerating or is highly maneuverable.

In two-dimensions, the continuous-time dynamic model of the target vehicle can be written in state-space form as

\[
\begin{bmatrix}
\dot{p}_N(t) \\
\dot{v}_N(t) \\
\dot{p}_E(t) \\
\dot{v}_E(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p_N(t) \\
v_N(t) \\
p_E(t) \\
v_E(t)
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\omega_{v_N}(t) \\
\omega_{v_E}(t)
\end{bmatrix}
\]

(3.7)

where \( \omega_{v_N} \) and \( \omega_{v_E} \) are assumed to be uncorrelated, zero-mean, Gaussian, white noise scalars

\[
E\{\omega_{v_N}(t)\} = 0
\]

(3.8)

\[
E\{\omega_{v_N}(t)\omega_{v_N}^T(t)\} = Q_{WN} \delta(t - \tau)
\]

(3.9)

\[
E\{\omega_{v_E}(t)\omega_{v_E}^T(t)\} = Q_{WE} \delta(t - \tau)
\]

(3.10)

\[
E\{\omega_{v_N}(t)\omega_{v_E}^T(t)\} = 0
\]

(3.11)

\[
E\{\omega_{v_E}(t)\omega_{v_N}^T(t)\} = 0
\]

(3.12)

For application with discrete-time filters, the continuous-time dynamic model must be discretized and written as its equivalent discrete-time dynamic model. The discrete equivalent of the matrix \( A \) can be determined by taking the inverse Laplace transform of the matrix \((sI - A)^{-1}\)

\[
A_d = \mathcal{L}^{-1}\{(sI - A)^{-1}\} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}
\]

(3.13)

where \( T \) refers to a sampling interval. The discrete equivalent of the power spectral density matrix is

\[
Q = \int_0^T \Phi(T, \tau) \Gamma_c Q_W \Gamma_c^T \Phi^T(T, \tau) d\tau
\]
\[
\begin{align*}
\int_0^T \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \tau & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} d\tau &= \int_0^T \begin{bmatrix} T^3/3 \\ T^2/2 \end{bmatrix} d\tau \\
&= Q_W \begin{bmatrix} T^3/3 \\ T^2/2 \end{bmatrix}
\end{align*}
\] (3.14)

In one-dimension, the discrete-time dynamic model of the target vehicle can be written in state-space form as
\[
\begin{bmatrix} p_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_k \\ v_k \end{bmatrix} + \omega_{vd,k}
\] (3.15)

where \(\omega_{vd,k}\) is assumed to be a zero-mean, Gaussian, white noise scalar
\[
\begin{align*}
E\{\omega_{vd,k}\} &= 0 \\
E\{\omega_{vd,k}^T \omega_{vd,k}\} &= Q_k
\end{align*}
\] (3.16) (3.17)

In two-dimensions, the discrete-time dynamic model of the target vehicle can be written in state-space form as
\[
\begin{bmatrix} p_{N,k+1} \\ v_{N,k+1} \\ p_{E,k+1} \\ v_{E,k+1} \end{bmatrix} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{N,k} \\ v_{N,k} \\ p_{E,k} \\ v_{E,k} \end{bmatrix} + \omega_{d,k}
\] (3.18)

or
\[
x_{k+1} = \Phi x_k + \omega_{d,k}
\] (3.19)

where \(\omega_{d}\) is assumed to be zero-mean, Gaussian, white noise process
\[
\begin{align*}
E\{\omega_{d,k}\} &= 0 \\
E\{\omega_{d,k}^T \omega_{d,k}\} &= Q_d
\end{align*}
\] (3.20) (3.21)

\[
Q_{d,k} = \begin{bmatrix} Q_{WN}T^3/3 & Q_{WN}T^2/2 & 0 & 0 \\ Q_{WN}T^2/2 & Q_{WN}T & 0 & 0 \\ 0 & 0 & Q_{WE}T^3/3 & Q_{WE}T^2/2 \\ 0 & 0 & Q_{WE}T^2/2 & Q_{WE}T \end{bmatrix}
\] (3.22)

We note that the pair \((\Phi, \sqrt{Q_d})\) is controllable.

### 3.3 Measurement Model

The measurement model of the stochastic system depends on the types of sensors located at each node. In general, sensors can provide measurements of the target vehicle’s position, velocity, range, bearing angle, and range rate. We select the measurement model based on the following assumptions:

- Each node has the same type of sensor.
- Each sensor measures the relative position of the target vehicle.

Based on these two assumptions, the measurement model for each sensor \(i\) is
\[
z_{Ti,k} = H_{si,k} x_k - x_{si} + v_{si,k}
\] (3.23)

where
\[
\begin{align*}
z_{Ti} &= \begin{bmatrix} p_{Ti,N} \\ T_{i,E} \end{bmatrix}^T \\
x_{si} &= \begin{bmatrix} p_{si,N} \\ p_{si,E} \end{bmatrix}^T \\
H_{si} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\end{align*}
\] (3.24) (3.25) (3.26)
The vector $x_{si}$ is uncorrelated with $x$ and $v_{si}$

$$E \left\{ (x_{si} - E \{x_{si}\}) (x_{si} - E \{x_{si}\})^T \right\} = R_{si,p} \tag{3.27}$$

$$E \left\{ (x_k - E \{x_k\}) (x_{si} - E \{x_{si}\})^T \right\} = 0 \tag{3.28}$$

$$E \left\{ (x_{si} - E \{x_{si}\}) v_{si,k}^T \right\} = 0 \tag{3.29}$$

where $R_{si,p}$ is the covariance matrix incorporating the sensor position uncertainty or, in other words, sensor localization errors. The vector $v_{si}$ is a zero-mean, Gaussian, white noise sequence uncorrelated with $x$ and $w_d$

$$E \{ v_{si,k} \} = 0 \tag{3.30}$$

$$E \{ v_{si,k} v_{si,k}^T \} = R_{si,w} + R_{si,TD,k} \tag{3.31}$$

$$E \{ (x_k - E \{x_k\}) v_{si,k}^T \} = 0 \tag{3.32}$$

$$E \{ w_{d,j} v_{si,k}^T \} = 0 \tag{3.33}$$

where $R_{si,w}$ is the covariance matrix incorporating the sensor wide band noise and $R_{si,TD}$ is the covariance matrix incorporating the sensor range errors to the target vehicle.

The covariance matrix of the innovation vector is

$$E \left\{ (z_{T_i,k} - E \{z_{T_i,k}\}) \right\} = E \left\{ [H_{si,k} x_k - x_{si} + v_{si,k} - (H_{si,k} E \{x_k\} - E \{x_{si}\})] \right\} \right\}^T$$

$$= E \left\{ [H_{si,k} (x_k - E \{x_k\}) - (x_{si} - E \{x_{si}\}) + v_{si,k}] \right\} \right\}^T$$

$$= E \left\{ [H_{si,k} (x_k - E \{x_k\})] \right\} \right\}^T + R_{si,k} \tag{3.34}$$

where $R_{si}$ refers to the total measurement uncertainty of each sensor $i$

$$R_{si,k} = R_{si,w} + R_{si,TD,k} + R_{si,p} \tag{3.35}$$

We note that pair $(\Phi, H_{si})$ is observable.
Chapter 4

Kalman Filter Design

In this chapter, we design the Kalman filter for the MSCT system. We make the following assumptions about the sensor measurement time:

- The measurement time synchronization error of sensor $i$, for all $i = 1, \ldots, N$, is 0.
- The measurement update frequency of the Kalman filter is equal to the sampling frequency of the sensors.

In Section 4.1, we design the Kalman filter for a MSCT system without timing errors. In Section 4.2, we design the Kalman filter for a MSCT system with timing errors.

4.1 Measurements without Timing Errors

If all sensor measurements at time $t_k$ arrive at the CPL at time $t_k$, then the following algorithm can be used to track the target vehicle using measurements from sensor $i$.

4.1.1 Initialize the filter

\[
\hat{x}_{0|0} = E\{x_0\} \tag{4.1}
\]
\[
P_{0|0} = E\{(x_0 - \hat{x}_{0|0})(x_0 - \hat{x}_{0|0})^T\} \tag{4.2}
\]

4.1.2 Time Update

\[
\hat{x}_{k+1|k} = \Phi \hat{x}_{k|k} \tag{4.3}
\]
\[
P_{k+1|k} = \Phi P_{k|k}\Phi^T + Q_d \tag{4.4}
\]

4.1.3 Measurement Update

\[
\hat{z}_{T_i,k+1|k} = H_{si} \hat{x}_{k+1|k} - \hat{x}_{si} \tag{4.5}
\]
\[
\nu_{T_i,k+1} = z_{T_i,k+1} - \hat{z}_{T_i,k+1|k} \tag{4.6}
\]
\[
K_{si,k+1} = P_{k+1|k}H_{si}^T\left(R_{si,k+1} + H_{si}P_{k+1|k}H_{si}^T\right)^{-1} \tag{4.7}
\]
\[
\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{si,k+1}\nu_{si,k+1} \tag{4.8}
\]
\[
P_{k+1|k+1} = P_{k+1|k} - H_{si}K_{si,k+1}P_{k+1|k} \tag{4.9}
\]
4.2 Measurements with Timing Errors

If a measurement taken by sensor \( i \) at time \( t_{k-l} \), where \( l \geq 1 \) is a positive integer, arrives at the CPL at time \( t_k \), then the algorithm in Section 4.1 must be modified to use the delayed measurement to update the statistics of the estimated state vector at time \( t_k \). We make the following assumption about the timing error of the MSCT system:

- The CPL receives measurements with an unknown, but bounded, time delay.

We use the following general approach to modify the algorithm for each sensor \( i \), for all \( i = 1, \ldots, N \).

1. compute the 1-step measurement model and vector that have the same effect on the statistics of the estimated state vector as all the measurement models and vectors within the time interval \( [t_{k-l+1}, t_k) \)

\[
 z_{si,eq,k} = H_{si,eq,k}x_k + v_{si,eq,k} \tag{4.10}
\]

\[
 E\{v_{si,eq,k}\} = 0 \tag{4.11}
\]

\[
 E\{v_{si,eq,k}v^T_{si,eq,k}\} = R_{si,eq,k} \tag{4.12}
\]

We assume that \( v_{si,eq,k} \) is uncorrelated with all process noise vectors within this time interval. We will refer to equation 4.10 as the equivalent measurement model.

2. retrodict the statistics of the estimated state vector from time \( t_k \) to time \( t_{k-l} \)

3. use the measurement vector and its statistics from time \( t_{k-l} \) to update the estimated state mean vector and estimated state covariance matrix at time \( t_{k-l} \)

4. use the updated estimated state mean vector and state covariance matrix at time \( t_{k-l} \) to update the estimated state mean vector and estimated state covariance matrix at time \( t_k \)

The dynamic model from time \( t_{k-l} \) to time \( t_k \) can be generalized as

\[
x_k = \Phi_{k, k-l}x_{k-l} + w_{k, k-l} \tag{4.13}
\]

where \( \Phi_{k, k-l} = \Phi_{k, k-l+1} \cdots \Phi_{k-l+1, k-l} \) and \( w_{k, k-l} \) refers to the cumulative process noise vector from time \( t_{k-l} \) to time \( t_k \)

\[
w_{k, k-l} = \sum_{j=0}^{l-1} \Phi_{k, k-l+j+1}w_{k-l+j+1, k-l+j} \tag{4.14}
\]

\[
 E\{w_{k, k-l}\} = 0 \tag{4.15}
\]

\[
 E\{w_{k, k-l}w^T_{k, k-l}\} = \sum_{j=0}^{l-1} \Phi_{k, k-l+j+1}Q_{d, k-l+j} \Phi^T_{k, k-l+j+1} \tag{4.16}
\]

We define the following expected values to clarify notation

\[
 \hat{x}_{k|k} = E\{x_k|Z(k)\} \tag{4.17}
\]

\[
 P_{k|k} = E\{(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T|Z(k)\} \tag{4.18}
\]

\[
 \hat{x}_{k|k-l} = E\{x_k|Z(k), z_{k-l}\} \tag{4.19}
\]

\[
 P_{k|k-l} = E\{(x_k - \hat{x}_{k|k-l})(x_k - \hat{x}_{k|k-l})^T|Z(k), z_{k-l}\} \tag{4.20}
\]

where \( Z(k) \) refers to all measurements processed at time \( t_k \).
4.2.1 Equivalent Measurement Model

The equivalent measurement model can be computed as follows. Using the information filter form of the Kalman filter [AM], [CJ], [SIM], [STE]

\[ P_{\text{eq},k}^{-1} = P_{k|k}^{-1} = P_{k|k-1}^{-1} + H_{\text{eq},k}^T R_{\text{eq},k}^{-1} H_{\text{eq},k} \] (4.21)

If we select \( H_{\text{eq},k} = I \), then

\[ R_{\text{eq},k}^{-1} = P_{k|k}^{-1} - P_{k|k-1}^{-1} \] (4.22)

The information filter form of the Kalman gain matrix is

\[ K_{\text{eq},k} = P_{k|k} H_{\text{eq},k}^T R_{\text{eq},k}^{-1} (4.23) \]

The update equation of the estimated state mean vector is

\[ \hat{x}_{k|k} = \hat{x}_{\text{eq},k|k} = \hat{x}_{k|k-1} + K_{\text{eq},k} \left( z_{\text{eq},k} - H_{\text{eq},k} \hat{x}_{k|k-1} \right) \] (4.24)

The equivalent measurement mean vector is

\[ z_{\text{eq},k} = H_{\text{eq},k} \hat{x}_{k|k-1} + K_{\text{eq},k} \left( z_{\text{eq},k} - H_{\text{eq},k} \hat{x}_{k|k-1} \right) \] (4.25)

The equivalent innovation mean vector and covariance matrix are

\[ \nu_{\text{eq},k} = z_{\text{eq},k} - H_{\text{eq},k} \hat{x}_{k|k-1} = K_{\text{eq},k} \left( \nu_{\text{eq},k} \right) \] (4.26)

\[ S_{\text{eq},k} = R_{\text{eq},k} + H_{\text{eq},k} P_{k|k-1} H_{\text{eq},k}^T \] (4.27)

Equations 4.25 - 4.27 are the equivalent measurement model and its statistics.

4.2.2 Delayed Measurement Kalman Filter

The following algorithm can be used to track the target vehicle using a delayed or asynchronous measurement vector from sensor \( i \) [BAR1], [BAR2]:

- retrodict the estimated state mean vector to time \( t_{k-1} \) from time \( t_k \)

\[ x_{k-1} = \Phi_{k,k-1}^{-1} x_{k-1} \]

\[ \hat{x}_{k-1|k} = E \{ x_{k-1|k} \} \]

\[ \Phi_{k,k-1}^{-1} \left( \hat{x}_{k-1|k} - Q_{d,k,k-1} H_{\text{eq},k}^T S_{\text{eq},k}^{-1} \nu_{\text{eq},k} \right) \] (4.28)

where we note that, in equation 4.28, the expected value of the process noise vector is not zero because of the retrodiction.

- compute the covariance matrix of the retrodicted process noise vector

\[ P_{w,k,k-1|k} = E \left( (w_{k-1} - E(w_{k-1})) (w_{k-1} - E(w_{k-1}))^T | Z(k) \right) \]

\[ = Q_{d,k,k-1} - Q_{d,k,k-1} H_{\text{eq},k}^T S_{\text{eq},k}^{-1} H_{\text{eq},k} Q_{d,k,k-1} \] (4.29)

- compute the cross-correlation matrix of the retrodicted state vector and the process noise vector

\[ P_{xw,k,k-1|k} = E \left( (x_{k} - E(x_{k})) (w_{k-1} - E(w_{k-1}))^T | Z(k) \right) \]

\[ = Q_{d,k,k-1} - P_{k,k-1} H_{\text{eq},k}^T S_{\text{eq},k}^{-1} H_{\text{eq},k} Q_{d,k,k-1} \] (4.30)
• compute the covariance matrix of the retrodicted state vector

\[ P_{k-l|k} = E \left\{ (x_{k-l} - E\{x_{k-l}\}) (x_{k-l} - E\{x_{k-l}\})^T | Z(k) \right\} \]

\[ = \Phi_{k-l}^{-1} \left( P_{k|k} - P_{xw,k,k-l|k} - P_{xw,k,k-l|k}^T + P_{w,k,k-l|k} \right) \Phi_{k-l}^{-T} \] (4.31)

• compute the covariance matrix of the measurement vector at time \( t_{k-l} \)

\[ S_{si,k-l} = R_{si,k-l} + H_{si,k-l} P_{k-l|k} H_{si,k-l}^T \] (4.32)

• compute the covariance matrix between the estimated state vector at time \( t_k \) and the measurement vector at time \( t_{k-l} \)

\[ P_{xz,k-l|k} = E \left\{ (x_k - E\{x_k\}) (z_{k-l} - E\{z_{k-l}\})^T | Z(k) \right\} \]

\[ = (P_{k|k} - P_{xw,k,k-l|k}) \Phi_{k-l}^{-T} H_{si,k-l} \] (4.33)

• use the delayed measurement vector at time \( t_{k-l} \) to update the estimated state mean vector and covariance matrix at time \( t_k \)

\[ \hat{x}_{k|k} = \hat{x}_{k|k} + P_{xz,k-l|k} S_{si,k-l}^{-1} (z_{si,k-l} - H_{si,k-l} \hat{x}_{k-l|k}) \] (4.34)

\[ P_{k|k} = P_{k|k} - P_{xz,k-l|k} S_{si,k-l}^{-1} P_{xz,k-l|k} \] (4.35)

We make the following observations about this algorithm.

1. Time delayed or asynchronous measurements are used to update the estimates of the state mean vector and state covariance matrix using the same approach.

2. This algorithm can be used for measurements with an arbitrary time delay.

3. During a measurement update, sensor measurements with the latest time stamps are processed first.
Chapter 5

Tracking Filter Performance

In this chapter, we evaluate the performance of the tracking filters designed for the MSCT system. We consider a sensor network consisting of four sensors located at positions

- Sensor 1: (2m,1m)
- Sensor 2: (4m,6m)
- Sensor 3: (-5m,-3m)
- Sensor 4: (-5m,6m)

in the N-E plane and three sensor operating scenarios, Figure 5.1. In the first scenario, or scenario A, measurements from all four sensors are used to track the target vehicle. We assume that the CPL receives sensor data packets from all sensor nodes without transmission delay. In the second scenario, or scenario B, measurements from Sensor 1 are used to track the target vehicle. We assume that the CPL receives sensor data packets from the sensor node without transmission delay. In the third scenario, or scenario C, measurements from Sensors 1 and 2 are used to track the target vehicle. We assume that the CPL receives sensor data packets form sensor node 1 without transmission delay and that the CPL receives sensor data packets from sensor node 2 with transmission delay.

For the three scenarios, the target vehicle is assumed to travel with a constant velocity of 2m/s at a constant heading angle of 45°. The starting position of the vehicle is assumed to be (-10m,-10m). The sensor measurements were simulated using the measurement models given in Chapter 3. In this application, the sensors measure the relative position of the target vehicle with a sampling frequency of 2Hz. Each sensor was assumed to have wide band noise of 0.1m (rms) in each axis so that $R_{si,w} = (0.1m)^2 I_2$. The sensor range errors were simulated by normalizing the true range between the target vehicle and sensor using the range between the initial position of the target vehicle and the sensor. We assume that there are no sensor localization errors so that $R_{si,p} = 0$.

The Kalman filters for all three scenarios were simulated using the system matrices outlined in Chapter 3 and the algorithms outlined in Chapter 4. The statistics of the initial state vector for all simulations were selected as

\[
\bar{x}_0 = \begin{bmatrix} -10m & 0m/s & -10m & 0m/s \end{bmatrix}^T \quad (5.1)
\]

\[
P_0 = 10 \times \text{diag} \begin{bmatrix} 1m^2 & 1m^2/s^2 & 1m^2 & 1m^2/s^2 \end{bmatrix} \quad (5.2)
\]

The time update rate for all simulations was 10Hz and, thus, the sampling rate for the matrices of the dynamic model was $T = 0.1s$. The measurement update rate was selected as the sensor sampling frequency or 2Hz.

Figures 5.2 and 5.3 show the tracking filter performance for scenario A. Figures 5.4 and 5.5 show the tracking filter performance for scenario B. In Figures 5.2 and 5.4, the estimated position (mean) vector is represented by solid lines whereas the true position vector is represented by dashed lines. In Figures 5.3 and 5.5, the position error vector, or position bias error, is represented by solid lines whereas the $1 - \sigma$...
estimation error bands are represented by dashed lines. We note that the $1 - \sigma$ estimation error bands are the standard deviations of the estimated position vector or, in other words, the square roots of the diagonal elements of the estimated state covariance matrix.

Figures 5.2 and 5.3 show that the estimated position vector converged to within its $1 - \sigma$ estimation error bands within 3.5s of filter initialization. Furthermore, the position bias error is less than $\pm 0.2m$ in each axis after filter convergence. The $1 - \sigma$ error estimation bands decrease as the target vehicle travels toward the sensors and these bands increase as the target vehicle travels away from the sensors. This effect is caused by the sensor ranging errors which depend on the range between the target vehicle and the sensors.

Figures 5.4 and 5.5 show that the estimated position vector converged to within its $1 - \sigma$ estimation error bands within 6s of filter initialization. Furthermore, the position bias error is less than $\pm 0.3m$ in each axis after filter convergence. As with scenario A, the $1 - \sigma$ error estimation bands decrease as the target vehicle travels toward the sensors and these bands increase as the target vehicle travels away from the sensors.

Figures 5.6 and 5.7 show the tracking filter performance for scenario C. In this scenario, the measurements from Sensor 2 are assumed to have a constant time delay as the target vehicle completes its trajectory. We assumed six different transmission delays ranging from 1 measurement update (MU) to 6 MUs. Figure 5.6 shows the position bias errors and Figure 5.7 shows the $1 - \sigma$ estimation error bands.

These figures show that as the transmission delay increases, both the position bias errors and the $1 - \sigma$ estimation error bands increase. The convergence time is shorter for all six transmission delay cases of scenario C as compared to scenario B. However, these figures also show that incorporating delayed measurements can temporarily increase both the position bias error and the $1 - \sigma$ estimation error bands until other measurements arrive at the CPL.
Figure 5.2: MSCT Tracking Performance - Position: Sensors 1, 2, 3, and 4
Figure 5.3: MSCT Tracking Performance - Position Bias Errors: Sensors 1, 2, 3, and 4
Figure 5.4: MSCT Tracking Performance - Position: Sensor 1
Figure 5.5: MSCT Tracking Performance - Position Bias Errors: Sensor 1
Figure 5.6: MSCT Tracking Performance - Position Bias Errors: Sensors 1 and 2

Delay 1 MU: blue
Delay 2 MU: black
Delay 3 MU: red
Delay 4 MU: cyan
Delay 5 MU: magenta
Delay 6 MU: green
Figure 5.7: MSCT Tracking Performance - $1 - \sigma$ Estimation Error Bands: Sensors 1 and 2

Delay 1 MU: blue
Delay 2 MU: black
Delay 3 MU: red
Delay 4 MU: cyan
Delay 5 MU: magenta
Delay 6 MU: green
Chapter 6

Summary

In this paper, we have designed two tracking filters for the MSCT system. The first filter incorporates measurements from sensors assuming that all measurements are received at the CPL without transmission delay. The second filter incorporates measurements from sensors assuming that the measurements are received at the CPL with arbitrary, but bounded, transmission delay. The performance results show that the tracking filters were able to compute bounded position bias error, bounded variance estimates of the target vehicle’s position vector for all three scenarios.

While the delayed measurement Kalman filter extends the performance of the hybrid-tracking filter designed in [ASYCS], this tracking filter has several limitations and, thus, extensions to the tracking filter are still required. First, the delayed measurement Kalman filter does not provide analytical bounds on the position bias error. Second, the actual performance of the delayed measurement Kalman filter can only be analyzed using Monte Carlo simulations where the configuration of the sensor network, the transmission time delays of sensor data packets to the CPL, and the target vehicle’s trajectory are used as input parameters to the simulations.

The two previously designed tracking filters will be extended by designing an $H_{\infty}$ filter. These filters are designed by specifically trying to minimize the bias error of the estimated state mean vector as a function of the various input parameters to the stochastic system. Therefore, the $H_{\infty}$ filter will provide the tools to analytically bound the position bias error as a function of the sensor network configuration, the transmission time delays of the sensor data packets to the CPL, and the target vehicle’s trajectory.
Chapter 7

References


