On The Study of Analogue Network Coding For Multi-Pair, Bidirectional Relay Channels

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Abstract—We consider a scenario where multiple pairs of users exchange information within pair with the help of a dedicated multi-antenna relay. The co-channel interference among multiple pairs is addressed in this paper. We propose a spectrally efficient protocol that uses the principal concept of network coding in mixing two data streams originating from the same user pair, coupled with the spatial multiplexing of the data streams originating from different user pairs. The key feature of the proposed protocol is that it enables both the relay and the users to participate in interference cancellation. Simulation and the analytical results justify that the ergodic capacity and the outage probability of the proposed protocol outperforms existing AF based schemes.

I. INTRODUCTION

Bidirectional relaying is a promising technique to enhance the throughput in wireless networks by reducing the channel resources used in the information exchange between users. Bidirectional relaying schemes such as strategies proposed in [1] based on decode-and-forward (DF) relaying, analogue network coding [2] based on amplify-and-forward (AF) relaying, physical network coding [3] based on estimate-and-forward (EF) relaying, etc are able to complete the two way information passing in only two phases. In the first phase, two users transmit simultaneously in the same channel to the relay. In the second phase, the relay forwards the processed mixture to the users and each user uses the knowledge of his previously transmitted message known as self-interference to decode the new message from his partner. The operation of mixing the messages with different origins at intermediate node coupled with the use of side information to decode mixture at the destination is an exhibit of network coding principal.

Attracted by the benefits of multi-antenna in enhancing the system capacity and reliability, bidirectional relaying has been generalised to the multi-antenna case. [4] generalises the DF based bidirectional relaying to multi-antenna setting using multiple access (MAC) and broadcast (BC) capacity regions. [5] proposes an AF based protocol which uses zero-forcing beamformer to eliminate the co-channel interference between users. However, no network coding principle is used to mix the data streams from two users. The special case where only the relay is equipped with multi-antenna is considered in [6]. The beamformers based on AF in [6] is designed to maximise the sum rate of single pair of users. On the other hand, the extension of the bidirectional relaying to multi-pair introduces the problem of multi-user interference. [7] proposes a scheme for CDMA system where each pair of users share a common spreading code as a means to reduce the multi-user interference. The proposed demodulate-and-forward based scheme in [7] uses multi-user receiver which requires high computational complexity at the relay. The suboptimal AF based scheme proposed in [7] suffers from poor BER performance when the number of users is low, because of noise domination. [8] proposes a scheme with DF relay for narrowband system which uses block diagonalisation to mitigate the interference caused by multi-pair. However, this scheme requires higher complexity for decoding/encoding at the relay if compared to the AF based scheme.

In this paper, we look into the multi-pair bidirectional relaying scenario with multi-antenna relay using AF based relaying. We propose a spectrally efficient two-phase bidirectional relaying protocol that utilises the principal concept of network coding in mixing two data streams originating from the same user pair coupled with the spatial multiplexing of the data streams originating from different user pairs. The key feature of the proposed network coding with spatial multiplexing (NC-SM) protocol is to allow both the users and the relay to participate in interference cancellation, where the relay eliminates the interference caused by multi-pair while each user eliminates the self-interference. We evaluate the performance of the proposed protocol using information theoretical metrics, namely the ergodic capacity and the outage probability and compare it with existing AF based bidirectional relaying schemes. We derive the analytical upper and lower bounds for the ergodic capacity using high SNR approximation. The simulation results verify the capacity bounds and show that the ergodic capacity of the proposed NC-SM protocol outperforms all existing AF based schemes. In the case where the relay has less antenna than the total number of single-antenna users, the proposed scheme is shown to achieve highest multiplexing gain among all existing AF based schemes. The simulation result also shows that the proposed protocol achieves better outage performance if compared to the comparable AF based multi-user zero-forcing scheme, when the number of antennas at the relay is at least the total number of single-antenna users.

This paper is organised as follows. The proposed protocol and comparable schemes are described in section II. The ergodic capacity bounds are presented in section III. In section IV, the numerical results on the ergodic capacity and the outage probability are discussed. The last section concludes the paper.
II. PROPOSED PROTOCOL DESCRIPTION

Consider a case where there are $M$ pairs of users exchanging information with their partners, such that each user is only interested in his partner’s message. A single dedicated relay is used to assist the information exchange between all user pairs. Each user has a single antenna while the dedicated relay has $N_r \geq 2M - 1$ antennas. We assume the symmetric case, where all nodes subject to unit average power constraint and has same channel statistics. All channels undergo i.i.d. quasi-static Rayleigh fading and channel reciprocity is assumed. The receiver is corrupted by circularly symmetric additive white Gaussian noise with distribution $CN(0, \sigma^2)$. Half duplex constraint is assumed throughout the paper and it is realised using the time division duplex. Every user knows his and his partner’s user-to-relay channel state information (CSI) while the relay has the CSI of all user-to-relay links.

The proposed protocol can be divided into two time slots. 

1) First time slot: During the first time slot, $M$ pairs of user nodes transmit simultaneously in the same channel with unit power. The relay will observe a mixture of all messages from the user nodes, which can be expressed as

$$\mathbf{r} = \sum_{m=1,i=2m-1,j=2m}^{M} \mathbf{h}_i \mathbf{x}_i + \mathbf{h}_j \mathbf{x}_j + \mathbf{n},$$

(1)

where the column vector $\mathbf{h}_i$ with size $N_r \times 1$ represents the channel from the user $i$ to the relay, the scalar $x_i$ is the message transmitted from user $i$, $\mathbf{n}$ with size $N_r \times 1$ denotes the noise vector observed by the relay. We use $(i,j)$ to represent the pair of user $i$ and user $j$ which exchange information with each other. Denote matrix $\mathbf{H}_{i,j} = [ \mathbf{h}_i \hspace{1em} \mathbf{h}_j ]$ as the combined channel from pair $(i,j)$ to the relay and $\mathbf{x}_{i,j} = [ x_i \hspace{1em} x_j ]^T$ as the message vector of pair $(i,j)$ and $[ ]^T$ denotes the transpose operation. We define the multi-pair interference channel for user pair $(i,j)$, $\mathbf{H}_{i,j} = [ \mathbf{H}_{1,2} \hspace{1em} \ldots \hspace{1em} \mathbf{H}_{i-2,j-2} \hspace{1em} \mathbf{H}_{i+2,j+2} \hspace{1em} \ldots \hspace{1em} \mathbf{H}_{2M-1,2M} ]$ with dimension $N_r \times 2(M-1)$, as the co-channel interference seen by user pair $(i,j)$.

2) Second time slot: During the second time slot, the dedicated relay broadcasts the linearly processed observations, i.e. $\mathbf{F} \mathbf{r}$, where $\mathbf{F}$ is the beamforming matrix. The joint receive and transmit beamforming structure is discussed in the following subsection.

A. Joint receive and transmit beamformer

The key idea is to allow both the users and the relay to participate in interference cancellation. The relay eliminates the multi-pair interference while the users removes the self interference. The receive beamforming matrix $\mathbf{W}_R$ and transmit beamforming matrix $\mathbf{W}_T$ are directly cascaded, yielding the following beamforming structure at the relay

$$\mathbf{F} = \mathbf{W}_T \mathbf{A} \mathbf{W}_R,$$

(2)

where the receive beamforming matrix $\mathbf{W}_R = [ \mathbf{w}_{R(1,2)} \hspace{1em} \ldots \hspace{1em} \mathbf{w}_{R(2M-1,2M)} ]^T$ has dimension $M \times N_r$ with its row vector $\mathbf{w}_{R(i,j)}^T$ as the receive beamforming vector for pair $(i,j)$, the transmit beamforming matrix $\mathbf{W}_T = [ \mathbf{w}_{T(1,2)} \hspace{1em} \ldots \hspace{1em} \mathbf{w}_{T(2M-1,2M)} ]$ has dimension $N_r \times M$ with its column vector $\mathbf{w}_{T(i,j)}$ as transmit beamforming vector for user pair $(i,j)$, while the diagonal matrix $\mathbf{A} = \text{diagonal} [ \alpha_{1,2} \hspace{1em} \ldots \hspace{1em} \alpha_{2M-1,2M} ]$ with dimension $M \times M$ is the power allocation matrix with scalar $\alpha_{i,j}$ as the power allocation factor for pair $(i,j)$. To enable the relay to eliminate the multi-pair interference for pair $(i,j)$, the receive beamforming vector is designed to lie in the left null space of the multi-pair interference channel, i.e. $\mathbf{w}_{R(i,j)}^T = \text{left null space}(\mathbf{H}_{i,j})$, such that the zero-forcing criterion $\mathbf{w}_{R(i,j)}^T \mathbf{H}_{i,j} = 0$ can be satisfied for all pair $(i,j)$. The left null space vectors exists only when the following dimension condition is satisfied [9], $N_r > \max \{ \text{rank}(\mathbf{H}_{1,2}), \ldots, \text{rank}(\mathbf{H}_{2M-1,2M}) \}$. This condition is satisfied when we choose $N_r \geq 2M - 1$. On the other hand, the relay transmitter makes sure that each user receives only the mixture containing his partner’s message and his own message (self-interference). In other words, each user pair shares a single beam. This is achieved when the transmit beamforming vector is designed to stay in the null space of the multi-pair interference channel, $\mathbf{w}_{T(i,j)}^T = \text{null space}(\mathbf{H}_{i,j})$ such that the zero-forcing criterion $\mathbf{H}_{i,j}^T \mathbf{w}_{T(i,j)} = 0$ is satisfied for any pair $(i,j)$. Due to channel reciprocity, it is easily seen that $\mathbf{w}_{T(i,j)}^T = \mathbf{w}_{R(i,j)}$.

For notation convenience, we drop the subscript of $T$ and $R$, and denote the beamforming vector for pair $(i,j)$, simply as $\mathbf{w}_{i,j}$.

The transmission from the relay is subjected to unit average power constraint. Since $\mathbf{w}_{i,j}$ is unit vector, the power constraint can be expressed as

$$\sum_{m=1,i=2m-1,j=2m}^{M} \alpha_{i,j}^2 ( ||\mathbf{w}_{i,j}\mathbf{w}_{i,j}^T\mathbf{H}_{i,j}||_F^2 + \sigma^2) \leq 1,$$

(3)

where $|| ||_F$ denotes the Frobenius norm and $\alpha_{i,j}$ is the power allocation factor for pair $(i,j)$. Since we are interested only in the high SNR performance, equal power allocation across $M$ data streams is sufficient. The equation above is satisfied in equality by choosing $\alpha_{i,j} = \frac{1}{\sqrt{M}} \frac{1}{\sqrt{||\mathbf{w}_{i,j}\mathbf{w}_{i,j}^T\mathbf{H}_{i,j}||_F^2 + \sigma^2}}$.

The signal received by user $i$ at the second time slot is

$$y_i = \alpha_{i,j} \mathbf{h}_i^T \mathbf{w}_{i,j} \mathbf{w}_{i,j}^T ( \mathbf{h}_i x_i + \mathbf{h}_j x_j + \mathbf{n} ) + n_i,$$

(4)

where $n_i$ is the receiver noise observed by user $i$. Since user $i$ has the knowledge of the self-interference, $x_i$, and the knowledge of the effective channel $\mathbf{h}_i^T \mathbf{w}_{i,j} \mathbf{w}_{i,j}^T \mathbf{H}_{i,j}$, he can decode the new message $x_j$ by subtracting the self-interference from the received mixture, where the principal of network coding is applied. It is important to notice that the effective channels of the $i$th user $\mathbf{h}_i^T \mathbf{w}_{i,j} \mathbf{w}_{i,j}^T \mathbf{h}_i$ and his partner $\mathbf{h}_i^T \mathbf{w}_{i,j} \mathbf{w}_{i,j}^T \mathbf{h}_j$ are scalars, which can be obtained easily with low overhead via training sequence or dedicated feedback links from the relay. Assuming Gaussian channel coding, the mutual information at user $i$ can be expressed as

$$I_i = \frac{1}{2} \log_2 \left( 1 + \frac{\alpha_{i,j}^2 ||\mathbf{h}_i^T \mathbf{w}_{i,j} \mathbf{w}_{i,j}^T \mathbf{h}_j||^2}{\sigma^2 (\alpha_{i,j}^2 ||\mathbf{h}_i^T \mathbf{w}_{i,j} \mathbf{w}_{i,j}^T||_F^2 + 1)} \right),$$

(5)
where \( ||.|| \) denotes the Euclidean norm, \(|.|\) denotes the absolute value. The pre-log factor reflects the two time slots used to complete the information exchange.

**B. Beamforming vector selection for the case \( N_r > 2M - 1 \)**

When the relay has more antennas than the minimum required antennas, i.e. \( N_r > 2M - 1 \), the nullity of the null space of the multi-pair interference channel \( H_{ij}^T \) becomes \( N_r - 2(M - 1) \), which is greater than 1. We propose to select the null space vector which maximises the sum-rate of each user pair. Denote the \( k \)th null space vector for pair \((i, j)\) as \( w_{i,j}(k) \) and the corresponding power allocation factor as \( \alpha_{i,j}(k) = \frac{1}{\sqrt{M}} \frac{1}{\sqrt{||w_{i,j}(k)H_{ij}||^2 + \sigma^2}} \). We have the following null space vector selection criterion for user pair \((i, j)\).

\[
\arg \max_{k=1,\ldots,N_r-2(M-1)} I_i(k) + I_j(k),
\]

where \( I_i(k) = \frac{1}{2} \log_2 \left( 1 + \frac{\alpha_{i,j}(k)^2 ||h_i^T w_{i,j}(k) w_{j,i}(k) h_j||^2}{\sigma^2 (\alpha_{i,j}(k)^2 ||w_{i,j}(k)w_{j,i}(k)||^2 + 1)} \right) \), and \( I_j(k) = \frac{1}{2} \log_2 \left( 1 + \frac{\alpha_{j,i}(k)^2 ||h_j^T w_{j,i}(k) w_{i,j}(k) h_i||^2}{\sigma^2 (\alpha_{j,i}(k)^2 ||w_{j,i}(k)w_{i,j}(k)||^2 + 1)} \right) \).

The best null space vector, denoted as \( w_{i,j}(k^*) \) is able to maximise the sum-rate of user pair \((i, j)\) and is used as the receive and transmit beamforming vectors. The sum-rate of user pair \((i, j)\) is used instead of the individual rate because the best null space vector, \( w_{i,j}(k^*) \) affects both user \( i \) and user \( j \) simultaneously.

**C. Baseline Schemes**

Three baseline schemes with AF based relay will be used for comparison, namely the pure AF scheme, maximal ratio reception and transmission (MRR-MRT) [6] and zero-forcing [5]. To enable fair comparison, the relay assumes the same average power constraint and the same number of antennas \( N_r \) as the proposed scheme. When \( N_r = 2M - 1 \), these existing two-phase bidirectional relaying schemes for single user pair can only be extended to multi-user pairs using time sharing between pairs. \(^1\) In general, the mutual information of the baseline schemes can be written as

\[
I_{\text{baseline}} = \frac{1}{2M} \log_2 \left( 1 + \frac{\alpha^2 ||h_i^T W h_i||^2}{\sigma^2 (\alpha^2 ||h_i^T W||^2 + 1)} \right),
\]  

where the power normalisation factor \( \alpha \) and the beamforming weighting matrix \( W \) depends on the specific scheme. The pre-log of \( 1/2M \) reflects the total time slots needed for \( M \) pairs of nodes to complete the information exchange. In the pure AF scheme, the relay forwards the power normalised observation to the target user pair without any linear processing at the relay. In the MRR-MRT scheme [6], the relay broadcasts a channel matched and power normalised observation to the target user pair. In the zero-forcing scheme [5], the relay uses conventional zero-forcing to separate the received mixture from the user pair into two orthogonal data streams and use zero-forcing to direct the orthogonal data streams to the intended destinations so that the destinations see no interference. This scheme is completely different from the proposed NC-SM scheme because it separates the data streams originating from the same user pair while the proposed NC-SM scheme preserves the mixture from the same user pair and orthogonalises the data streams originating from different user pairs. When \( N_r > 2M - 1 \), all baseline schemes except the zero-forcing scheme use time sharing between pairs. The zero-forcing scheme is able to orthogonalise the data streams from all user pairs and complete the information exchange in two time slots, since the relay has enough antenna for the null-space projection operation. In this case, the pre-log in (7) is \( \frac{1}{2} \). It will be shown in the numerical results section that the proposed NC-SM protocol still outperforms the existing zero-forcing scheme when \( N_r > 2M - 1 \).

**III. Ergodic Capacity Analysis**

This section provides the analytical results of the ergodic capacity for the case when \( N_r = 2M - 1 \), where significant multiplexing gain improvement is achieved. Since we consider the symmetrical channels, i.e. all the users have the same channel statistics, it is sufficient to study the single-user ergodic capacity. Define the single-user ergodic capacity as the per user long term data rate a system can support, which can be expressed as

\[
C_{\text{erg}} = \int_0^\infty f_{\hat{I}}(\hat{I}) d\hat{I},
\]

where \( f_{\hat{I}}(\hat{I}) \) is the probability density function (PDF) of the mutual information \( \hat{I} \). Recall the mutual information of the proposed protocol in (5). In the following, we drop the subscript \( w \) for simplicity. First, we look into the properties of the variables in \( \hat{I} \). Represent \( W = WW^H \) where \( ||.||^H \) denotes the Hermitian transpose operation. The matrix \( W \) is a positive semi-definite Hermitian matrix and more importantly, \( W \) carries the idempotent property, i.e. \( WW^H = W \). Using the associative property of matrix multiplication, the numerator \( ||h_i^T W h_i||^2 \) in (5) can be split into \( ||h_i^T W||^2 ||w_i||^2 \). We can write each of the absolute value square as vector multiplication, i.e. \( ||h_i^T W||^2 = h_i^T W h_i^* \), where \( ||.|^* \) denotes the conjugate operation. Using the idempotent property of \( W \), we can represent \( h_i^T W h_i^* = h_i^T WW^H h_i = ||h_i^T W||^2 \). Following similar approach, we can write \( ||W h_i||^2 = ||W^T h_i||^2 \), where \( W^T = w^T w^H \). Define the auxiliary variables \( x = \frac{xy}{(M+1)x + My + M} \) and \( y = \frac{1}{2} ||W^T h_i||^2 \). After some algebraic manipulations, we can rewrite the mutual information in (5) as following

\[
\hat{I} = \frac{1}{2} \log_2 \left( 1 + \frac{xy}{(M+1)x + My + M} \right).
\]  

Note that \( ||W^T h_i||^2 = ||h_i^T W||^2 \). Defining auxiliary variable \( z = \frac{xy}{(M+1)x + My + M} \), we have the following bounds for the cumulative distribution function (CDF) for variable \( z \).

**Lemma 1:** The CDF of variable \( z \) can be bounded as

\[
1 - \exp\left(-2(M+1)^2 z^2\right) \leq F_z(z) \leq 1 - \exp\left(-2(M+1)^2 z^2 - 2\sigma^2 \sqrt{M(M+1)z^2 + Mz}\right).
\]  

\(^1\) Although simultaneous channel access by all users is possible under DF based relaying subjects to MAC and BC capacity regions, it is beyond the scope of this paper to compare AF based schemes with DF based schemes.
The density function \( f_z(z) \) derived from the following formula

\[
1 - \exp \left( -(2M+1)\sigma^2 z - 2\sigma^2 \sqrt{M(M+1)z^2 + Mz} \right) \\
\approx 1 - \exp \left( -( (2M+1)\sigma^2 + 2 \sqrt{M(M+1)\sigma^2} ) z \right). \tag{11}
\]

Recall that the mutual information is a function of \( z \), such that \( I(z) = \frac{1}{2} \log_2 (1 + z) \). According to section 7.4.9 in [11], the expected value of \( I \) (which is the ergodic capacity) can be derived from the following formula

\[
C_{\text{erg}} = \int_0^\infty I(z) f_z(z) dz. \tag{12}
\]

The density function \( f_z(z) \) can be obtained by differentiating \( F_z(z) \) over \( z \), i.e. \( f_z(z) = \frac{dF_z(z)}{dz} \). We have the following theorem for the ergodic capacity.

**Theorem 1:** In the scenario where \( M \) pairs of single-antenna users are assisted by a dedicated relay with \( N_r = 2M-1 \) antennas, the ergodic capacity for each user of the proposed NC-SM protocol can be upper and lower bounded using high SNR approximation, as follows,

\[
c \left( 2M + 1 + 2 \sqrt{M(M+1)} \right) \leq C_{\text{erg}} \leq c \left( 2M + 1 \right) , \tag{13}
\]

where function \( c(x) = \frac{1}{2} \exp \left( x\sigma^2 \right) \log_2 \left( \frac{1}{\pi\sigma^2 \exp(\gamma)} \right) \) and \( \gamma \) is the Euler constant.

**Proof:** Refer to [10].

**Remark 1:** For fixed number of user pairs, \( M \), we can write the lower bound at high SNR, i.e. \( \frac{1}{\sigma^2} \to \infty \), as \( C_{\text{erg,lower}} \approx \frac{1}{2} \log_2 \left( \frac{1}{\sigma^2} \right) = \frac{1}{2} \log_2 \left( \left( 2M + 1 + 2 \sqrt{M(M+1)} \right) \exp(\gamma) \right) \).

The first term is a function of SNR while the second term is a constant independent of the SNR. Therefore, for every 3dB increase in SNR, there is a 0.5 bits/s/Hz increment on the ergodic capacity. In other words, the achievable per user multiplexing gain is \( \frac{1}{2} \), which is independent of the number of user pairs. The second term captures two effects. First, the power loss due to the null space projection operation used in the beamformer and second, the equal power sharing among \( M \) pairs of users. As the number of user pairs increases, the transmission power at the relay is shared by more users, and hence each user gets a smaller portion of the total power. This explains why the second term increases logarithmically with \( M \). The ergodic capacity decreases as the number of active user pairs increases, but the individual multiplexing gain is maintained.

**IV. NUMERICAL RESULTS**

In this section, we present Monte Carlo simulation results to validate the analytical result and assess the performance of the proposed protocol in comparison with existing AF based schemes in terms of single-user ergodic capacity and single-user outage probability.

Fig. 1 compares the ergodic capacity versus SNR of the proposed protocol and three baseline schemes as mentioned in subsection II-C, for the case when \( N_r = 2M-1 \). The analytical bounds for the ergodic capacity is included in the figure. The number of user pairs is fixed at \( M=2 \) while the number of antennas at the relay is fixed at \( N_r=3 \). It can be observed that from medium to high SNR, i.e. SNR>15dB, the ergodic capacity of the proposed NC-SM protocol outperforms all existing schemes. At SNR=30dB, gain of 36%, 45% and 75% are obtained by the proposed NC-SM scheme in comparison with the MRR-MRT, zero-forcing, and the pure AF schemes respectively. This reveals that allowing both the relay and the user to participate in the interference cancellation is better than interference cancellation at either relay (zero-forcing scheme) or the user (MRR-MRT and pure AF schemes). The proposed protocol also demonstrates that spatial multiplexing across different pairs coupled with network coding within same pair is spectrally more efficient. At low SNR, the diversity based scheme such as MRR-MRT performs better because the system is power limited rather than bandwidth limited in this region. Furthermore, from fig.1, the analytical upper and lower bounds approximate the slope of the simulated ergodic capacity very well from medium to high SNR, i.e. SNR>15dB. When the SNR=30dB, the gaps between the bounds and the simulation result is less than 0.26 bits/s/Hz. The slope of the ergodic capacity curve characterises the multiplexing gain. The steeper the slope, the higher the multiplexing gain is. It is obvious that the proposed NC-SM protocol delivers the highest multiplexing gain among all schemes.

Fig. 2 shows the ergodic capacity versus SNR for various schemes when \( N_r > 2M-1 \). In this simulation, the fixed parameters are \( M=2 \) and \( N_r=4 \). The main difference between fig. 2 and the previous fig. 1 is that the zero-forcing scheme has the same ergodic capacity slope as the proposed scheme. As discussed in subsection II-C, when the number of antennas at the relay is at least the total number of single-antenna users, the zero-forcing scheme uses spatial multiplexing instead of time sharing between pairs to complete the information exchange in two slots. All other baseline schemes still depend on time-sharing between pairs.
In the SNR range of interest, i.e. SNR>10dB, the proposed NC-SM scheme with beamforming vector selection always performs better than the zero-forcing scheme. To achieve ergodic capacity of 3 bits/s/Hz, the proposed scheme offers a power saving of around 2.5dB if compared to the zero-forcing scheme. This improvement is due to the additional diversity achieved by the beamforming vector selection in the proposed protocol. Enabling the user to participate in the self-interference cancellation results in smaller interference space seen by each user. This leads to the increase in the dimension of the null space, which allows the beamformer to choose best beamforming vector to maximise the performance. The diversity gain introduced by the proposed protocol is further justified from the outage probability versus SNR curves shown in Fig. 3. In this figure, the outage performance of the proposed NC-SM scheme with beamforming vector selection is compared with the existing zero-forcing scheme at various target data rate, R. The fixed parameters are M = 2 and N_r = 4. Generally, the proposed scheme achieves lower outage probability at all target data rate. The slope of the outage probability curve characterises the diversity gain. The proposed scheme has a higher diversity gain than existing multi-user zero-forcing scheme.

V. CONCLUSIONS

The proposed protocol of combining network coding and spatial multiplexing that allows both the relay and users to participate in the cancellation of multi-pair interference and self-interference yields significant improvement in terms of ergodic capacity and outage performance. The developed analytical bounds for the ergodic capacity approximate the simulated ergodic capacity very well. Simulations show that the proposed protocol achieves higher ergodic capacity than comparable schemes while produces better outage performance if compared to existing multi-user zero-forcing scheme.

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